Spin density induced by electromagnetic waves in a two-dimensional electron gas with both Rashba and Dresselhaus spin-orbit coupling

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We consider the magnetic response of a two-dimensional electron gas with both Rashba and Dresselhaus spin-orbit coupling to a microwave excitation. We generalize the results of Shnirman and Martin [Europhys. Lett. **78**, 27001 (2007)], where pure Rashba coupling was studied. We observe that the microwave with the in-plane electric field and the out-of-plane magnetic field creates an out-of-plane spin polarization. The effect is more prominent in clean systems with resolved spin-orbit-split subbands. Considered as response to the microwave magnetic field, the spin-orbit contribution to the magnetization far exceeds the usual Zeeman contribution in the clean limit. The effect vanishes when the Rashba and the Dresselhaus couplings have equal strength.

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I. INTRODUCTION

The spin-orbit effects in semiconductors have a long history.[1–3] Several years ago, the interest in the subject was renewed by the proposal of the intrinsic spin-Hall effect,[4,5] i.e., a possibility to generate a spin current in systems with intrinsic spin-orbit coupling by applying electric field. Initially the effect was considered for conductors, e.g., for a two-dimensional electron gas (2DEG),[4] while now the attention of the community has mostly switched to the quantum spin-Hall effect in "topological insulators."[6,7]

It has also been realized that the concept of spin current is not as clear and useful as the concept of charge current.[8] Due to the lack of spin conservation the presence of spin current does not necessarily lead to spin accumulation, and vice versa. For example, it was concluded that in a 2DEG with the Rashba and the Dresselhaus spin-orbit interactions, the bulk spin current vanishes for constant and homogeneous electric field.[9–12] At the same time the spin accumulation at the edges of two-dimensional (2D) samples was observed in Refs. 13 and 14, igniting arguments whether the observed effect was intrinsic or extrinsic.

In Ref. 15 the out-of-plane spin polarization in a 2DEG with purely Rashba spin-orbit coupling was studied. The research was motivated by a wish to determine the spin polarization without using the concept of spin current and to avoid considering spin accumulation at edges. Thus, the idea was to create an out-of-plane inhomogeneous spin density *in the bulk* in response to a spatially modulated electric field. One can realize that for this purpose one needs a *transversal* electric field which necessitates a time dependence, i.e., microwaves. In other terms, one can consider the results of Ref. 15 as providing the spin response to a long wave (spatially homogeneous), out-of-plane, oscillating in time magnetic field. According to the Faraday induction law such a field creates spatially inhomogeneous, in-plane electric field which, in turn, creates the out-of-plane spin polarization. In the clean limit this orbital effect of the magnetic field greatly dominates over the spin polarization due to the direct Zeeman coupling.

In this Brief Report we generalize the results of Ref. 15 to the case where both Rashba and Dreselhaus interactions are present, which is experimentally more relevant than the limit considered in Ref. 15. The corresponding spin-orbit term in the Hamiltonian reads

$$H_{SO} = \alpha_R(-p_x\sigma_y + p_y\sigma_x) + \beta_D(p_x\sigma_x - p_y\sigma_y), \qquad (1)$$

where α_R and β_D are the strengths of the Rashba and Dresselhaus spin-orbit couplings, respectively. It is convenient to perform a $\pi/4$ rotation in both the momentum and the spin spaces, that is $p'_x = \frac{p_x+p_y}{\sqrt{2}}$ and $p'_y = \frac{-p_x+p_y}{\sqrt{2}}$, and $\sigma'_x = \frac{\sigma_x+\sigma_y}{\sqrt{2}}$ and $\sigma'_y = \frac{-\sigma_x+\sigma_y}{\sqrt{2}}$. In the new coordinates Hamiltonian (1) reads

$$H_{SO} = -(\alpha_R+\beta_D)p'_x\sigma'_y + (\alpha_R-\beta_D)p'_y\sigma'_x. \qquad (2)$$

In what follows we work in the rotated basis and omit the primes as well as use the units with $\hbar=1$.

Introducing the angle ϕ via $\boldsymbol{p}=|\boldsymbol{p}|(\cos\phi,\sin\phi)$ we obtain the energies of the two subbands given by $\epsilon^{\pm}(\boldsymbol{p}) = \frac{\boldsymbol{p}^2}{2m^*} \pm \gamma_\phi|\boldsymbol{p}|$, where m^* is the electron band mass, $\gamma_\phi \equiv \sqrt{(\alpha_R^2+\beta_D^2)(1+W\cos 2\phi)}$, and $W \equiv \frac{2\alpha_R\beta_D}{\alpha_R^2+\beta_D^2}$. For purely Rashba (Dresselhaus) coupling $W=0$, while in the case of equal coupling strengths $W=1$.

We consider a linearly polarized in-plane microwave field $\boldsymbol{A}=\boldsymbol{A}_0\exp(i\boldsymbol{q}\boldsymbol{r}-i\Omega t)$, where $\boldsymbol{A}_0=A_0(\cos\theta,\sin\theta,0)$ and $\boldsymbol{q}=q(\sin\theta,-\cos\theta,0)$. The signs are chosen so that for positive A_0 and q the vectors $\boldsymbol{q},\boldsymbol{A}_0,\boldsymbol{e}_z$ form a right-handed basis. We also recall the standard relations $\boldsymbol{E}=(i\Omega/c)\boldsymbol{A}$ and $\boldsymbol{B}=i\boldsymbol{q}\times\boldsymbol{A}$.

II. KINETIC EQUATION

The technique we use here is exactly the same as in Ref. 15. It is based on the standard Kubo linear-response theory in

Keldysh formulation.[16] This technique allows one to determine the dynamics of the charge and spin densities. In the dirty limit (to be defined below) it leads to diffusion equations for the spin and charge densities.[10,17,18] However, we do not restrict ourselves to this limit and obtain results valid both in the clean and in the dirty cases.

For integrity we remind the main elements of the working formalism. We employ the linear response, $H=H_0+H_1$, with

$$H_0=\frac{\boldsymbol{p}^2}{2m^*}+\boldsymbol{\eta p}+V_{\text{disorder}}, \tag{3}$$

where $\boldsymbol{\eta}=[-(\alpha_R+\beta_D)\sigma_y,(\alpha_R-\beta_D)\sigma_x]$, and

$$H_1=-\frac{e}{2c}\{\boldsymbol{v},\boldsymbol{A}\}_+-\frac{1}{2}g\mu_B\boldsymbol{B\sigma},$$

where $\boldsymbol{v}\equiv\frac{\boldsymbol{p}}{m^*}+\boldsymbol{\eta}$. The first term in H_1 is responsible for the orbital effect discussed in this Brief Report. The second term causes the usual (Pauli) spin polarization. We consider only the s-wave disorder scattering and introduce the inverse momentum relaxation time τ^{-1}, as well as the electronic density of states per spin $\nu=m^*/(2\pi)$.

The zeroth order in the $\boldsymbol{A}$ Green's function, G_0, contains the standard disorder broadening

$$G_0^R=\left(\frac{1}{2}+\frac{1}{2}\frac{\boldsymbol{\eta p}}{\gamma_\phi|\boldsymbol{p}|}\right)G_0^{R+}+\left(\frac{1}{2}-\frac{1}{2}\frac{\boldsymbol{\eta p}}{\gamma_\phi|\boldsymbol{p}|}\right)G_0^{R-}, \tag{4}$$

where $G_0^{R\pm}(\boldsymbol{p},\omega)\equiv[\omega-\epsilon^{\pm}(\boldsymbol{p})+i/(2\tau)]^{-1}$. In equilibrium $G_0^K=h(\omega)(G_0^R-G_0^A)$, where $h(\omega)\equiv\tanh\frac{\omega-E_F}{2T}$.

Introducing the average spin-orbit band splitting $\Delta_F\equiv p_F\sqrt{\alpha_R^2+\beta_D^2}$, we can define three regimes: (i) "superclean" $\tau^{-1}<\Delta_F^2m^*/p_F^2=m^*(\alpha_R^2+\beta_D^2)$; (ii) clean $m^*(\alpha_R^2+\beta_D^2)<\tau^{-1}<\Delta_F$; (iii) dirty $\tau^{-1}>\Delta_F$. Our results below apply both in the clean and in the dirty regimes, but not in the "superclean" one, i.e., our results apply for $\tau^{-1}>\Delta_F^2m^*/p_F^2$.

Within the self-consistent Born approximation we find the linear in $\boldsymbol{A}$ correction to the Green's function, G_1. Establishing G_1 we can calculate any single-particle quantity, i.e., density or current. The Keldysh component G_1^K splits into two parts, $G_1^K=G_1^{K,\text{I}}+G_1^{K,\text{II}}$. The first part, $G_1^{K,\text{I}}$, corresponds to the retarded-advanced (R-A) combinations in the Kubo formula, while $G_1^{K,\text{II}}$ stands for the R-R and A-A combinations.[16] The spin-charge density matrix is defined as $\hat{\rho}=\frac{1}{2}n(\boldsymbol{q},\Omega)+\boldsymbol{s}(\boldsymbol{q},\Omega)\boldsymbol{\sigma}=\int\frac{d\omega}{2\pi}\int\frac{d^2p}{(2\pi)^2}[-iG_1^<]=-\frac{i}{2}\int\frac{d\omega}{2\pi}\int\frac{d^2p}{(2\pi)^2}(G_1^K-G_1^R+G_1^A)$, where $n(\boldsymbol{q},\Omega)$ is the charge density while $\boldsymbol{s}(\boldsymbol{q},\Omega)$ is the spin density. Accordingly, it splits as $\hat{\rho}=\hat{\rho}^{\text{I}}+\hat{\rho}^{\text{II}}$, where $\hat{\rho}^{\text{I}}=-\frac{i}{2}\int\frac{d\omega}{2\pi}\int\frac{d^2p}{(2\pi)^2}G_1^{K,\text{I}}$ and $\hat{\rho}^{\text{II}}=-\frac{i}{2}\int\frac{d\omega}{2\pi}\int\frac{d^2p}{(2\pi)^2}(G_1^{K,\text{II}}-G_1^R+G_1^A)$.

Thus, following Ref. 16 we obtain the linear-response relation

$$\frac{(1-I)}{\tau}\hat{\rho}^{\text{I}}=i\Omega\nu\tilde{I}\left[\frac{e\{\boldsymbol{vA}\}_+}{2c}+\frac{1}{2}g\mu_B\boldsymbol{B\sigma}\right], \tag{5}$$

where the functional $\tilde{I}$ is defined as

$$\tilde{I}[X(\boldsymbol{p})]=\frac{1}{m^*\tau}\int\frac{d^2p}{(2\pi)^2}G_0^R(\boldsymbol{p}+\boldsymbol{q}/2,E_F+\Omega/2)\cdot X(\boldsymbol{p})\times G_0^A(\boldsymbol{p}-\boldsymbol{q}/2,E_F-\Omega/2), \tag{6}$$

while I is a 4×4 matrix defined by its action on the four-vector $\hat{\rho}$ as $I\hat{\rho}=\tilde{I}[\hat{\rho}]$ (we just use the fact that $\hat{\rho}$ is independent of $\boldsymbol{p}$ to represent the functional $\tilde{I}[\hat{\rho}]$ as a product of a 4×4 matrix I and a vector $\hat{\rho}$).

The second contribution to the density, $\hat{\rho}^{\text{II}}$, is given by $\hat{\rho}^{\text{II}}=\frac{1}{2}g\nu\mu_B\boldsymbol{B\sigma}$. Thus, the total density response follows from

$$\frac{(1-I)}{\tau}\hat{\rho}=i\Omega\nu\tilde{I}\left[\frac{e\{\boldsymbol{vA}\}_+}{2c}\right]+\frac{[1-(1-i\Omega\tau)I]}{2\tau}g\nu\mu_B\boldsymbol{B\sigma}. \tag{7}$$

We allow for arbitrary external frequency Ω, including $\Omega>\tau^{-1}$. However, we limit ourselves to the experimentally relevant regime $v_F|\boldsymbol{q}|\ll\tau^{-1}$. For arbitrary values of $\boldsymbol{q}$ the spin and charge response functions to the longitudinal fields have been recently calculated in Ref. 19.

We expand the matrix I in powers of $\boldsymbol{q}$, $I=I^{(0)}+I^{(1)}+\ldots$. In zeroth order in $\boldsymbol{q}$ the matrix I is diagonal, and its elements are given by

$$I_{00}^{(0)}=\frac{1}{a},\quad I_{zz}^{(0)}=\frac{a}{\sqrt{R}},$$

$$I_{xx}^{(0)}=\frac{(Q+D)-b^2(1+W)}{2a\sqrt{R}},$$

$$I_{yy}^{(0)}=\frac{(Q-D)-b^2(1-W)}{2a\sqrt{R}}, \tag{8}$$

where $R\equiv(a^2+b^2)^2-b^4W^2=[a^2+(1+W)b^2][a^2+(1-W)b^2]$, $D\equiv(a^2+b^2-\sqrt{R})/W$, and $Q\equiv(a^2+b^2+\sqrt{R})$. We have introduced $a\equiv1-i\Omega\tau$ and $b\equiv2\Delta_F\tau$. Analyzing the path of the complex function $R(\Omega)$ we conclude that for $\sqrt{R}$ we have to choose the branch cut along the positive semiaxis $R>0$.

For the part linear in $\boldsymbol{q}$, $I^{(1)}$, we obtain the following matrix elements:

$$I_{zx}^{(1)}=-I_{xz}^{(1)}=\frac{p_F|\boldsymbol{q}|\tau\sin\theta}{m^*}i_{zx}^{(1)},$$

$$I_{zy}^{(1)}=-I_{yz}^{(1)}=\frac{p_F|\boldsymbol{q}|\tau\cos\theta}{m^*}i_{zy}^{(1)}, \tag{9}$$

where

$$i_{zx}^{(1)}\equiv-\frac{iab\sqrt{\frac{1+W}{R}}}{a^2+b^2(1+W)},\quad i_{zy}^{(1)}\equiv\frac{iab\sqrt{\frac{1-W}{R}}}{a^2+b^2(1-W)}. \tag{10}$$

We have neglected terms mixing the charge density with the spin density since they are smaller by a factor of $m/(p_F^2\tau)\ll1$ than the spin-spin terms.

Expanding the right-hand side (RHS) of Eq. (7) in powers of $\boldsymbol{q}$ up to the linear order and neglecting again the charge-density term for the same reason as above we obtain an expression for the orbital term

$$i\Omega\nu\widetilde{I}\left[\frac{e\{\boldsymbol{vA}\}_+}{2c}\right]=C_{E,x}^{(0)}\sigma_x+C_{E,y}^{(0)}\sigma_y+C_{E,z}^{(1)}\sigma_z$$
$$=\frac{\nu e|\boldsymbol{E}|\sin\theta}{\tau p_F}c_{E,x}^{(0)}\sigma_x+\frac{\nu e|\boldsymbol{E}|\cos\theta}{\tau p_F}c_{E,y}^{(0)}\sigma_y+\frac{\nu e|\boldsymbol{E}||\boldsymbol{q}|}{m^*}c_{E,z}^{(1)}(\theta)\sigma_z. \quad (11)$$

The dimensionless coefficients contributing to the zeroth order in $\boldsymbol{q}$ are given by

$$c_{E,x}^{(0)}=-\frac{b(1+W)\sqrt{\frac{1-W}{R}}(b^2-D)}{4a},$$

$$c_{E,y}^{(0)}=\frac{b(1-W)\sqrt{\frac{1+W}{R}}(b^2+D)}{4a}, \quad (12)$$

while the coefficient linear in $\boldsymbol{q}$ is

$$c_{E,z}^{(1)}(\theta)=i\sqrt{1-W^2}\times\frac{[b^4(1-W^2)-a^4]b^2-(2Wa^2b^4+RD)\cos 2\theta}{4aR\sqrt{R}}. \quad (13)$$

Finally, the Zeeman term in the RHS of Eq. (7) reads

$$\frac{[1-(1-i\Omega\tau)I]}{2\tau}g\nu\mu_B\boldsymbol{B\sigma}=\frac{\nu g\mu_B}{2\tau}\sum_{j=x,y,z}[1-aI_{jj}^{(0)}]B_j\sigma_j. \quad (14)$$

Note that the magnetic terms ($\propto\boldsymbol{B}=i\boldsymbol{q}\times\boldsymbol{A}$) are already of the first order in $\boldsymbol{q}$.

We are now in a position to calculate the spin density. First we obtain the zeroth order in $\boldsymbol{q}$ contribution. It is given by

$$s_x^{(0)}=\frac{\tau}{1-I_{xx}^{(0)}}C_{E,x}^{(0)}=-\frac{\nu eE_y}{2p_F}\frac{b(1+W)\sqrt{1-W}(b^2-D)}{2a\sqrt{R}-(Q+D)+b^2(1+W)},$$

$$s_y^{(0)}=\frac{\tau}{1-I_{yy}^{(0)}}C_{E,y}^{(0)}=\frac{\nu eE_x}{2p_F}\frac{b(1-W)\sqrt{1+W}(b^2+D)}{2a\sqrt{R}-(Q-D)+b^2(1-W)},$$

$$s_z^{(0)}=0. \quad (15)$$

This is a generalization of the well-known result,[2,3] meaning that there is an in-plane spin polarization "perpendicular" to the applied electric field (cf. Ref. 20). For $W\neq 0$ it can only be perpendicular to $\boldsymbol{E}$ if the electric field directs along one of the main axes x or y.

Next we calculate the first-order orbital contribution. We obtain

$$s_z^{(1),\text{orbital}}=\frac{\tau}{1-I_{zz}^{(0)}}C_{E,z}^{(1)}-\frac{\tau}{1-I_{zz}^{(0)}}\left(-\frac{I_{zx}^{(1)}}{\tau}\right)\frac{\tau}{1-I_{xx}^{(0)}}C_{E,x}^{(0)}-\frac{\tau}{1-I_{zz}^{(0)}}\left(-\frac{I_{zy}^{(1)}}{\tau}\right)\frac{\tau}{1-I_{yy}^{(0)}}C_{E,y}^{(0)}$$
$$=\frac{\tau C_{E,z}^{(1)}+I_{zx}^{(1)}s_x^{(0)}+I_{zy}^{(1)}s_y^{(0)}}{1-I_{zz}^{(0)}}=\frac{\nu e|\boldsymbol{E}||\boldsymbol{q}|\tau}{m^*}\frac{1}{1-I_{zz}^{(0)}}\times\left[c_{E,z}^{(1)}(\theta)+\frac{i_{zx}^{(1)}c_{E,x}^{(0)}\sin^2\theta}{1-I_{xx}^{(0)}}+\frac{i_{zy}^{(1)}c_{E,y}^{(0)}\cos^2\theta}{1-I_{yy}^{(0)}}\right]$$
$$=\nu\mu_B B_z\left(\frac{m_e}{m^*}\right)\frac{2\Omega\tau}{1-I_{zz}^{(0)}}\times\left[c_{E,z}^{(1)}(\theta)+\frac{i_{zx}^{(1)}c_{E,x}^{(0)}\sin^2\theta}{1-I_{xx}^{(0)}}+\frac{i_{zy}^{(1)}c_{E,y}^{(0)}\cos^2\theta}{1-I_{yy}^{(0)}}\right], \quad (16)$$

where $\mu_B\equiv e/(2m_ec)$ and m_e is the bare electron mass.

Finally, for the Zeeman term we obtain

$$s_z^{(1),\text{Zeeman}}=\frac{1-aI_{zz}^{(0)}}{1-I_{zz}^{(0)}}\cdot\frac{\nu g\mu_B}{2}B_z. \quad (17)$$

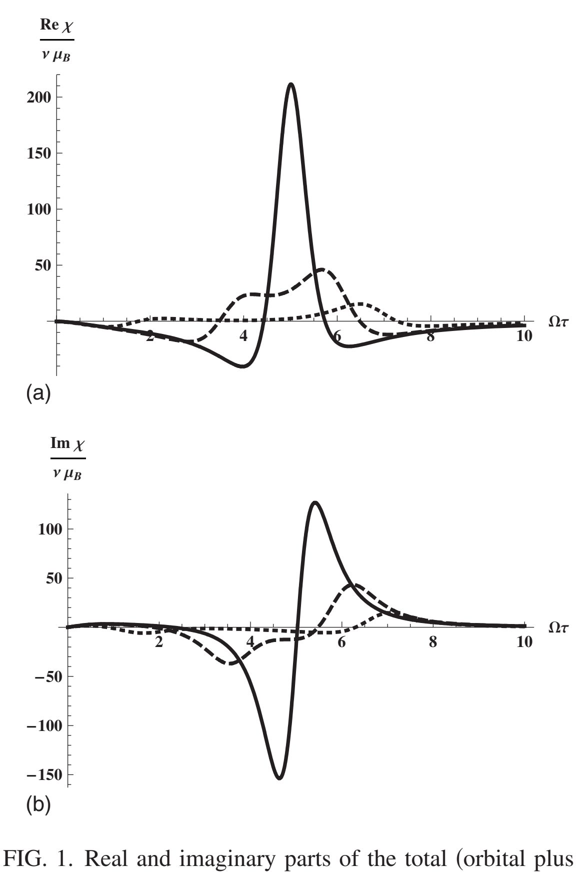

FIG. 1. Real and imaginary parts of the total (orbital plus Zeeman) spin susceptibility $\chi\equiv s_z/B_z$ for $2\Delta_F\tau=5$. Solid lines: $W=0.1$, dashed lines: $W=0.5$, dotted lines: $W=0.9$. Parameters assumed as in GaAs: $m_e/m^*\approx 15$, $g=-0.44$. The results are plotted for $\theta=\pi/4$ which also corresponds to the averaged over θ susceptibility.

III. DISCUSSION

We observe that both parts of the out-of-plane spin polarization s_z, i.e., the orbital part given by Eq. (16) and the Zeeman part given by Eq. (17) can be regarded as linear response to the out-of-plane magnetic field B_z. Thus our analysis amounts to a calculation of the susceptibility $\chi(\omega,\boldsymbol{q})$ so that $s_z=\chi B_z$.

We observe that in the clean limit, i.e., for $\Delta_F > \tau^{-1}$, the orbital susceptibility greatly dominates over the Zeeman one. As one can see in Figs. 1 and 2, for experimentally relevant parameters, the susceptibility χ exceeds the Pauli susceptibility by a factor of order hundreds.

In the vicinity of $W=0$ we reproduce the results of Ref. 15 and the susceptibility is peaked around $\Omega=2\Delta_F$. For W substantially different from zero a double-peak structure develops with the positions $\Omega=2\sqrt{1\pm W}\Delta_F=2p_F|\alpha_R\pm\beta_D|$. The pole singularity present at $W=0$ splits in this case into two square-root singularities corresponding to zeros of function $R(\Omega)$ at $\Omega=2\sqrt{1\pm W}\Delta_F-i/\tau$. One or another peak is emphasized depending on the angle θ as seen in Fig. 2.

We obtained our results for a microwave with a given direction of the wave-vector $\boldsymbol{q}$, i.e., for a given angle θ. The most obvious way to observe the orbital contribution to the spin susceptibility would be by applying a homogeneous oscillating magnetic field B_z, e.g., by putting the sample into a magnetic coil. Such a field corresponds to an equal superposition of plane waves with all possible wave vectors $\boldsymbol{q}$ lying in the xy plane. To obtain the orbital spin response in this case one should just average over θ, i.e., substitute $\langle\cos 2\theta\rangle=0$ and $\langle\cos^2\theta\rangle=\langle\sin^2\theta\rangle=1/2$. This is what we plot in Fig. 1. Note, that for $W=0$, i.e., for pure Rashba or Dresselhaus coupling, the response is θ independent and the averaging brings nothing new. On the other hand, when the two couplings are of comparable strength, the susceptibility strongly depends on θ (see Fig. 2) and averaging over θ can change the result considerably. At $W=1$ the orbital susceptibility vanishes.

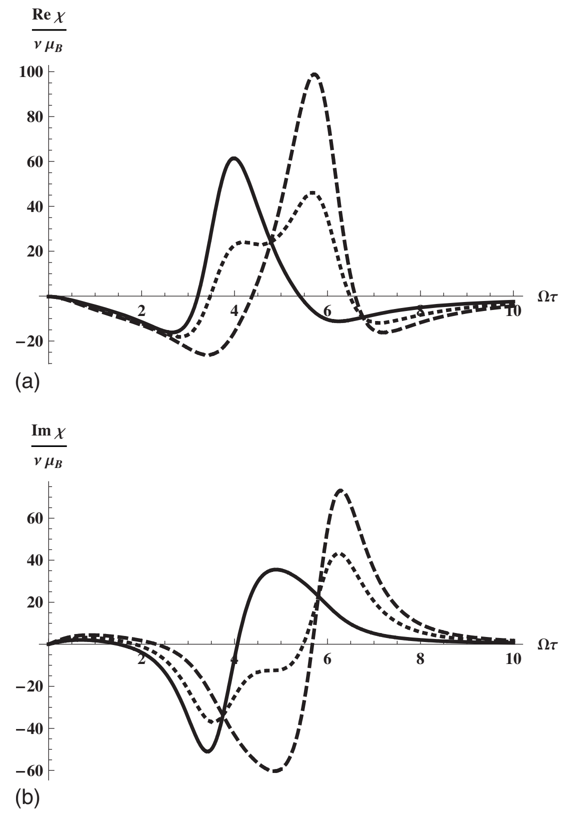

FIG. 2. Real and imaginary parts of the total (orbital plus Zeeman) spin susceptibility $\chi\equiv s_z/B_z$ for $2\Delta_F\tau=5$ and $W=0.5$. Solid lines: $\theta=0$, dashed lines: $\theta=\pi/2$, dotted lines: $\theta=\pi/4$. Parameters assumed as in GaAs: $m_e/m^*\approx 15$, $g=-0.44$.

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